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The objective of this research was to develop tools leading to effective control strategies for tailless fighter aircraft. Emphasis was placed on the development of control algorithms that yield robust performance in the presence of actuator magnitude and rate limits and that account for the interaction of a pilot with the airframe and control system. The reults of this research are documented in the twenty-two papers that are listed in this report. These papers present the development of numerous new analysis tools for nonlinear control systems with an emphasis on disturbance attenuation and sampled-data control These algorithms were applied to the manual flight control problem for open-loop unstable fighter aircraft.

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ANDREW R. TEEL
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FINAL REPORT

ROBUST NONLINEAR CONTROL OF TAILLESS FIGHTER AIRCRAFT

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Summary

The objective of the research supported by the Air Force Office for Scientific Research under Grant No. AF/F49620-98-1-0087, which ran from December 1, 1997 through November 30, 1998, was to develop tools leading to effective control strategies for tailless fighter aircraft. Emphasis was placed on the development of control algorithms that yield robust performance in the presence of actuator magnitude and rate limits and that account for the interaction of a pilot with the airframe and control system. The results of our research are documented in the twenty-two papers that are listed in the next section. In these papers we have developed numerous new analysis tools for nonlinear control systems with an emphasis on disturbance attenuation and sampled-data control. Further, we have extended the algorithms that we previously developed for control (especially anti-windup control) in the presence of actuator rate and magnitude limits. Moreover, we have applied these algorithms to the manual flight control problem for open-loop unstable fighter aircraft.

1 Research Publications

The research supported by this grant resulted in 5 journal papers to appear, 3 journal papers submitted and 1 journal paper in preparation. It also resulted in 1 book chapter and 12 refereed conference papers published, to appear and submitted. These papers are listed below.

1. A.R. Teel and L. Praly. "On assigning the derivative of a disturbance attenuation control Lyapunov function", *Mathematics of Control, Signals and Systems*, to appear, 1999.

2. A.R. Teel, J. Peuteman and D. Aeyels. "Global asymptotic stability for the averaged implies semi-global practical asymptotic stability for the actual", *Systems & Control Letters*, to appear, 1999.

3. A.R. Teel. "Asymptotic convergence from $\mathcal{L}_p$ stability", *IEEE Transactions on Automatic Control*, to appear, 1999.

4. A.R. Teel. "Anti-windup for exponentially unstable linear systems", *International Journal of Robust and Nonlinear Control*, special issue on "Control problems with constraints", to appear, 1999.

5. D. Nesic, A.R. Teel and E.D. Sontag. "Formulas relating $\mathcal{KL}$ stability estimates of discrete-time and sampled-data nonlinear systems", *Systems & Control Letters*, to appear, 1999.

6. R. Sepulchre, M. Arcak and A.R. Teel. "Trading the stability of finite zeros for global stabilization of nonlinear cascade systems", Submitted to *IEEE Transactions on Automatic Control*, 1998.

7. K. Ezal, P.V. Kokotovic, A.R. Teel and T. Basar. "Disturbance attenuating output-feedback control of nonlinear systems with local optimality", Submitted, 1998.

8. T.I. Fossen, A. Loria and A.R. Teel. "A theorem for UGAS and ULES of nonautonomous systems: robust control of mechanical systems and ships", Submitted to the *International Journal of Robust and Nonlinear Control*, 1998.

9. A.R. Teel and L. Praly. "A smooth Lyapunov function from a class-$\mathcal{KL}$ estimate involving two positive semidefinite functions", in preparation.

Book Chapters

10. A. Isidori, A.R. Teel and L. Praly, Dynamic UCO controllers and semiglobal stabilization of uncertain nonminimum phase systems by output feedback, *New Trends in Nonlinear Observer Design*, Springer-Verlag, 1999, to appear.

Refereed Conference Publications

11. A.R. Teel. "A nonlinear control viewpoint on anti-windup and related problems", *Preprints of the 4th IFAC Nonlinear Control Systems Design Symposium*, 1998, Enschede, the Netherlands, pp. 115–120.

12. A.R. Teel and L. Praly. "On assigning the derivative of a disturbance attenuation clf", *Proceedings of the 37th IEEE Conference on Decision and Control*, Tampa, FL, 1998.

13. A.R. Teel, J. Peuteman and D. Aeyels. "Global asymptotic stability for the averaged implies semi-global practical asymptotic stability for the actual", *Proceedings of the 37th IEEE Conference on Decision and Control*, Tampa, FL, 1998.

14. A.R. Teel, D. Nesic and P. Kokotovic. "A note on input-to-state stability of sampled-data nonlinear systems", *Proceedings of the 37th IEEE Conference on Decision and Control*, Tampa, FL, 1998.

15. C. Barbu, R. Reginatto, L. Zaccarian and A.R. Teel. "Anti-windup design for manual flight control", *1999 American Control Conference*, San Diego, CA, to appear.

16. D. Nesic, A.R. Teel and E.D. Sontag. "On stability and input-to-state stability $\mathcal{KL}$ estimates of discrete-time and sampled-data nonlinear systems", *1999 American Control Conference*, San Diego, CA, to appear.

17. D. Nesic, A.R. Teel and P. Kokotovic. "On the design of a controller based on the discrete-time approximation of the nonlinear plant model", *1999 American Control Conference*, San Diego, CA, to appear.

18. K. Ezal, P.V. Kokotovic, A.R. Teel and T. Basar. "Disturbance attenuating output-feedback control of nonlinear systems with local optimality", *1999 American Control Conference*, San Diego, CA, to appear.

19. A. Loria, T. I. Fossen, and A.R. Teel "UGAS and ULES of Nonautonomous Systems: Applications to Integral Control of Ships and Manipulators". In *Proceedings of the 5th European Control Conference*, Karlsruhe, Germany, 1999, to appear.

20. A. Loria, E. Panteley, and A.R. Teel. "A new notion of persistency-of-excitation for UGAS of NLTV systems: application to stabilization of nonholonomic systems", In *Proceedings of the 5th European Control Conference*, Karlsruhe, Germany, 1999, to appear.

21. A.R. Teel and L. Praly, "Results on converse Lyapunov functions from class-$\mathcal{KL}$-estimates", Submitted to the *38th IEEE Conference on Decision and Control*.

22. D. Nesic and A.R. Teel, "On averaging and the input-to-state stability property", Submitted to the *38th IEEE Conference on Decision and Control*.

2 Research Accomplishments

2.1 Analysis tools for nonlinear control systems

2.1.1 New averaging results

In [1] and [2] we established new Lyapunov stability analysis tools that permit deducing stability, respectively input-to-state stability, from the behavior of the Lyapunov function evaluated along trajectories at sampling instances. In turn we applied these tools to establish new results on averaging for nonlinear differential equations that are potentially very useful in vibrational control, for example. In [1] we showed showed that if the average of a time-varying system is globally asymptotically stable then the time-varying system is semiglobally practically asymptotically stable as the time variations are made arbitrarily fast. The novelty of this result is its global nature and the fact that no exponential stability assumptions are required. Later, we extended this result to systems with exogenous disturbances, showing that if the average of a time-varying system with disturbances is input-to-state stable (which is a certain $\mathcal{L}_\infty$ stability notion) then the time-varying system is semiglobally practically input-to-state stable as the time-variations are made arbitrarily fast.

We now describe, in more detail, the input-to-state stability result. We consider differential equations of the form

$$\dot{x}(t) = f\left(\frac{t}{\epsilon}, x(t), w(t)\right) \tag{1}$$

and we assume that f is locally Lipschitz in x and w, uniformly in t and $f(t, 0, 0)$ is bounded. A locally Lipschitz function $f_{wa} : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^n$ is said to be the *weak average* of $f(t, x, w)$ if there exist $\beta \in \mathcal{KL}$ and $T^* > 0$ such that $\forall T \geq T^*, \forall t \geq 0$ we have

$$\left| f_{wa}(x, w) - \frac{1}{T} \int_t^{t+T} f(s, x, w)ds \right| \leq \beta(\max\{|x|, |w|\}, T).$$

Theorem 1 *If f has the weak average f_{wa} and there exist a smooth function $V : \mathbb{R}^n \to \mathbb{R}_{\geq 0}, \alpha_1, \alpha_2, \alpha_3 \in \mathcal{K}_\infty, \gamma \in \mathcal{G}$ such that $\alpha_1(|x|) \leq V(x) \leq \alpha_2(|x|)$ and*

$$|x| \geq \gamma(|w|) \quad \Longrightarrow \quad \frac{\partial V}{\partial x} f_{wa}(x, w) \leq -\alpha_3(|x|)$$

then there exists $\beta \in \mathcal{KL}$ and, given any strictly positive real numbers $\Omega_x, \Omega_w, \Omega_{\dot{w}}, \nu$, there exists $\epsilon^* > 0$ such that, $\forall \epsilon \in (0, \epsilon^*)$, the solutions of (1) satisfy:

$$|x(t)| \leq \max\{\beta(|x(t_0)|, t - t_0), \alpha_1^{-1} \circ \alpha_2 \circ \gamma(\|w\|_\infty)\} + \nu, \quad \forall t \geq t_0 \geq 0,$$

whenever w is absolutely continuous, $|x(t_0)| \leq \Omega_x, \|w\|_\infty \leq \Omega_w, \|\dot{w}\|_\infty \leq \Omega_{\dot{w}}.$

2.1.2 Sampled-data analysis tools

In the papers [3] and [4] we have analyzed nonlinear sampled-data systems controlled by, respectively, algorithms based on continuous-time methods and discrete-time methods. In [3] we showed, for general nonlinear systems, that sampling sufficiently fast an input-to-state stabilizing continuous time control law results in semiglobal practical input-to-state stability as the sampling period is decreased to zero. The result is proved by writing the closed-loop sampled-data system as a functional differential equation and appealing to recent results (established by this investigator under a previous AFOSR grant) on sufficient Razumikhin-type conditions for input-to-state stability for functional differential equations. The main results of [3] provide a tool for analyzing the input-to-state stability of high performance nonlinear control laws implemented with sample and hold devices, including modern flight control systems.

The main results of [3] are as follows. We consider a locally Lipschitz control system $\dot{x} = f(x, u, w)$ where w represents exogenous disturbances. We suppose we have developed a local Lipschitz feedback law $u(x)$ that induces input-to-state stability with respect to the exogenous disturbance w. Now we consider the effect of sampling the state at discrete-time instances and holding the input at the corresponding value until the next sampling instance. In this case the control system can be written as the functional differential equation

$$\dot{x}(t) = f(x(t), u(x(t - \tau(t))), w) \tag{2}$$

where $\tau(t)$ is a sawtooth wave that is periodic with sampling period T. In [3] we proved the following:

Theorem 2 *If there exist a smooth function $V : \mathbb{R}^n \to \mathbb{R}_{\geq 0}, \alpha_1, \alpha_2, \alpha_3 \in \mathcal{K}_\infty, \gamma \in \mathcal{G}$ such that $\alpha_1(|x|) \leq V(x) \leq \alpha_2(|x|)$ and*

$$|x| \geq \gamma(|w|) \qquad \Longrightarrow \qquad \frac{\partial V}{\partial x} f(x, u(x), w) \leq -\alpha_3(|x|)$$

then there exists $\beta \in \mathcal{KL}$ and, given any strictly positive real numbers Ω_x, Ω_w, ν, there exists $T^* > 0$ such that, for all sampling periods $T \in (0, T^*)$, the solutions of (2) satisfy:

$$|x(t)| \leq \max\{\beta(|x(t_0)|, t - t_0), \alpha_1^{-1} \circ \alpha_2 \circ \gamma(\|w\|_\infty)\} + \nu, \quad \forall t \geq t_0 \geq 0,$$

whenever $|x(t_0)| \leq \Omega_x, \|w\|_\infty \leq \Omega_w$.

In [4] we analyzed nonlinear sampled-data systems controlled by discrete-time methods. In that paper, we pointed out that sufficiently fast sampling does not guarantee that discrete-time control laws that stabilize a discrete-time approximate model actually stabilize the sampled-data system. This is illustrated by the triple integrator $\dot{x}_1 = x_2$, $\dot{x}_2 = x_3$, $\dot{x}_3 = u$, its Euler approximate model

$$\begin{aligned} x_1(k+1) &= x_1(k) + T x_2(k) \\ x_2(k+1) &= x_2(k) + T x_3(k) \\ x_3(k+1) &= x_3(k) + T u(k), \end{aligned} \tag{3}$$

and a minimum-time dead-beat controller for the Euler discrete-time model given by

$$u(k) = \left(-\frac{x_1(k)}{T^3} - \frac{3x_2(k)}{T^2} - \frac{3x_3(k)}{T} \right).$$

(4)

The closed loop system consisting of (3) and (4) has all poles equal to zero for all $T > 0$ and hence this discrete-time Euler-based closed loop system is asymptotically stable for all $T > 0$. On the other hand, the closed loop system consisting of the exact discrete-time model of the triple integrator and controller (4) has a pole at ≈ -2.644 for all $T > 0$. Hence the closed-loop sampled-data control system is unstable for all $T > 0$. So we see that, to design a stabilizing controller using an approximate discrete-time model, it is not sufficient to design a stabilizer for an approximate discrete-time model of the plant for all $T > 0$. Extra conditions are needed.

In [4] we developed sufficient conditions which guarantee that this intuitively appealing approach to sampled-data control design actually works. We considered the difference equations corresponding to the exact discrete-time model of a locally Lipschitz differential equation and a discrete-time approximation, respectively:

$$x(k+1) = F_T^e(x(k), u(k)) \tag{5}$$
$$x(k+1) = F_T^a(x(k), u(k)) . \tag{6}$$

These models are parameterized by the sampling period T. We denote the state of the system (5) (resp. (6)) with the given controller $u(k) = u_T(x(k))$ at time step k and initial condition $x(0)$ as $x^e(k, x(0))$ (resp. $x^a(k, x(0))$). The family (u_T, F_T^a) is said to be a *higher order approximation for* (u_T, F_T^e) if there exists $r > 0$ and, for each bounded neighborhood B of the origin, there exist $K > 0, T^* > 0$ such that for all $T \in]0, T^*[$ and for all $x, z \in B$ we have

$$|F_T^e(x, u_T(x)) - F_T^a(z, u_T(z))| \le (1 + KT)|x - z| + KT^{r+1} .$$

In [4] we established the following result (the input-to-state stability version of these results is in preparation):

Theorem 3 *Let $\beta \in \mathcal{KL}$ and let N be a bounded neighborhood of the origin. If the family (u_T, F_T^a) is a higher order approximations for (u_T, F_T^e) and there exists $T^* > 0$ such that for each $T \in (0, T^*)$, we have*

$$|x^a(k, x(0))| \le \beta(|x(0)|, kT) \qquad \forall x(0) \in N , \forall k \ge 0$$

then for each $R > 0$ there exists $T^* > 0$ such that for each $T \in (0, T^*)$ we have

$$|x^e(k, x(0))| \le \beta(|x(0)|, kT) + R \qquad \forall x(0) \in N , \forall k \ge 0 .$$

In turn, we used the main results of [5] to draw the corresponding conclusions for the sampled-data control system.

2.1.3 Generalized converse Lyapunov functions

Converse Lyapunov function theory has been instrumental over the years in establishing robustness of nonlinear control algorithms. As generalized notions of stability are introduced into the nonlinear control and stability literature, there is a need to develop converse Lyapunov function results for these new stability notions, and in very general contexts. In [6], we made a very significant contribution in that direction. We studied very general differential inclusions so that we would have a tool that could be used to analyze the robustness of generalized stability induced by discontinuous control laws. Moreover, we assumed a fairly generic stability notion where one positive semidefinite function of the trajectories of the system is upper bounded by a class-$\mathcal{KL}$ function of a positive semidefinite function of the initial condition and the elapsed time. We showed that a smooth converse Lyapunov function for this type of stability exists if and only if the stability is robust in an appropriate sense. Then we gave some sufficient conditions for robustness that allowed us to recover most of the known results in converse Lyapunov function theory.

More specifically, we let $\omega_i : \mathcal{G} \to \mathbb{R}_{\geq 0}$, $i = 1, 2$, be continuous functions and we say that the differential inclusion $\dot{x} \in F(x)$ is $\mathcal{KL}$-stable with respect to (ω_1, ω_2) on $\mathcal{G}$ if it is forward complete on $\mathcal{G}$ and there exists $\beta \in \mathcal{KL}$ such that, for each $x \in \mathcal{G}$, all solutions $\phi(t, x)$ satisfy

$$\omega_1(\phi(t, x)) \leq \beta(\omega_2(x), t) \qquad \forall t \geq 0 .$$

This stability notion covers standard asymptotic stability concepts as well as asymptotic stability of closed sets and partial asymptotic stability.

We say that the differential inclusion $\dot{x} \in F(x)$ is *robustly $\mathcal{KL}$-stable with respect to* (ω_1, ω_2) *on* $\mathcal{G}$ if if there exists a continuous function $\delta : \mathcal{G} \to \mathbb{R}_{\geq 0}$ such that

1. $\{x\} + \delta(x)\overline{\mathcal{B}} \subset \mathcal{G}$;

2. the inclusion
$$\dot{x} \in F_{\delta(x)}(x) := \overline{\mathrm{co}}F(x + \delta(x)\overline{\mathcal{B}}) + \delta(x)\overline{\mathcal{B}} \tag{7}$$
   is $\mathcal{KL}$-stable with respect to (ω_1, ω_2) on $\mathcal{G}$;

3. $\delta(x) > 0$ for all $x \in \mathcal{G} \backslash \mathcal{A}_\delta$ where

$$\mathcal{A}_\delta := \left\{ \xi \in \mathcal{G} : \sup_{t \geq 0, \phi \in \mathcal{S}_\delta(\xi)} \omega_1(\phi(t, \xi)) = 0 \right\} \tag{8}$$

and where $\mathcal{S}_\delta(\cdot)$ represents the set of solutions to (7).

Finally, we characterize a converse Lyapunov function for $\mathcal{KL}$-stability with respect to (ω_1, ω_2) as follows: A function $V : \mathcal{G} \to \mathbb{R}_{\geq 0}$ is said to be a *smooth converse Lyapunov*

function for $\mathcal{KL}$-stability with respect to (ω_1, ω_2) on $\mathcal{G}$ for $F(x)$ if $V(x)$ is smooth on $\mathcal{G}$ and there exist class-$\mathcal{K}_\infty$ functions α_1, α_2 and α_3 such that, for all $x \in \mathcal{G}$,

$$\alpha_1(\omega_1(x)) \leq V(x) \leq \alpha_2(\omega_2(x)) \tag{9}$$

and

$$\max_{w \in F(x)} \langle \nabla V(x), w \rangle \leq -\alpha_3(V(x)) . \tag{10}$$

One of the main results of [6] is the following:

Theorem 4 *Let $\omega_i : \mathcal{G} \to \mathbb{R}_{\geq 0}$, $i = 1, 2$, be continuous and let $F(x)$ be upper semicontinuous and nonempty, compact and convex on $\mathcal{G}$. The following statements are equivalent:*

1. *The inclusion $\dot{x} \in F(x)$ is forward complete on $\mathcal{G}$ and there exists a smooth converse Lyapunov function for $\mathcal{KL}$-stability with respect to (ω_1, ω_2) on $\mathcal{G}$ for $F(x)$.*

2. *The inclusion $\dot{x} \in F(x)$ is robustly $\mathcal{KL}$-stable with respect to (ω_1, ω_2) on $\mathcal{G}$.*

We then went on to show, in [6], that if the set-valued map $F(x)$ is locally Lipschitz or $\omega_1 = \omega_2 =: \omega$ and backward finite escape times can be observed through ω then $\mathcal{KL}$-stability with respect to (ω_1, ω_2) is robust. This allowed us to recover most of the known theorems on the existence of converse Lyapunov functions.

2.1.4 Asymptotic Convergence and Uniform asymptotic stability

It is important in time-varying, nonlinear control systems to establish uniform asymptotic stability of the origin rather than only uniform stability plus convergence to the origin because the former (together with a Lipschitz condition, uniform in t, on the right-hand side of the differential equation) implies robustness with respect to disturbances while the latter does not. For some very common nonlinear control algorithms, e.g., adaptive backstepping and nonlinear PI control, it is not very straightforward to establish this uniform asymptotic stability (in a set of error coordinates) when trying to track time-varying reference signals. On the other hand, many tools are available for guaranteeing (nonuniform) convergence to zero of the tracking error. In [7] (see also [8]) we studied systems of the form

$$\begin{aligned} \dot{x}_1 &= h(x_1, t) + G(x, t)x_2 \\ \dot{x}_2 &= D(x, t) \end{aligned}$$

and provided useful conditions for guaranteeing uniform (global) asymptotic stability of the origin. These results were then applied to establish uniform global asymptotic stability in tracking error coordinates for mechanical systems and ships controlled via adaptive backstepping and nonlinear PI control algorithms. These results were further generalized in [9] and applied to stabilization of nonholonomic systems by continuous, time-varying feedback. We produced a related result in [10].

10

2.2 Nonlinear Control Design and Applications

2.2.1 Assigning the derivative of a disturbance attenuation clf

One of the most important problems in nonlinear control design is to induce a prescribed level of disturbance attenuation at the controlled output. A natural way to characterize a wide-class of disturbance attenuation results is through a dissipation inequality of the form

$$\dot{V}(x(t)) \leq \widetilde{\alpha}(x(t), d(t)) \tag{11}$$

where the derivative of the positive semidefinite function V is taken along trajectories $x(t)$ satisfy the closed-loop differential equation $\dot{x} = f_{cl}(x, d)$. The specific disturbance attenuation problem that is being solved, e.g., $\mathcal{L}_2$ disturbance attenuation or $\mathcal{L}_\infty$ disturbance attenuation, dictates the desired form for the function $\widetilde{\alpha}$. In [11], we considered control systems of the form

$$\dot{x} = f(x, d) + g(x)u \tag{12}$$

and, given a locally Lipschitz positive semidefinite function V and an arbitrary function $\widetilde{\alpha}$, looked at the problem of synthesizing a control $u(x)$ to induce the dissipation inequality (11). More specifically, we considered the problem where a function $\pi(x)$ was given so that the function[1] $L_{g(x)}V(x)\pi(x)$ is nonpositive and we want to find a feedback of the form $u(x) = \psi(x)\pi(x)$ where ψ is scalar, nonnegative and locally bounded so that the dissipation inequality (11) holds. In [11], we gave conditions, in terms of $V(x)$, $\pi(x)$, $f(x, d)$, $g(x)$ and $\widetilde{\alpha}$, that allowed us to synthesize a feedback law that induces the dissipation inequality (11). The result is as follows. We say that $V(x)$ is a *control Lyapunov function (clf) for the pair* $(\pi, \widetilde{\alpha})$ if $L_{g(x)}V(x)\pi(x)$ is nonpositive, $\omega(x)$ given by

$$\omega(x) := \sup_d \left\{ L_{f(x,d)}V(x) - \widetilde{\alpha}(x, d) \right\} \tag{13}$$

is well-defined, $\max\{0, \omega(x)\}$ is locally bounded, and, for $x \neq 0$,

$$\limsup_{z \to x} L_{g(z)}V(z)\pi(z) = 0 \qquad \Longrightarrow \qquad \limsup_{z \to x} \omega(z) < 0 . \tag{14}$$

We say $V(x)$ satisfies the *bounded control property (bcp) for the pair* $(\pi, \widetilde{\alpha})$ if there exist $\chi > 0$ and $\bar{v} \geq 0$ such that, with $\omega(x)$ defined in (13),

$$\omega(x) + \bar{v}L_{g(x)}V(x)\pi(x) \leq 0 \qquad \forall |x| \leq \chi . \tag{15}$$

The main result of [11] is the following:

Theorem 5 *If $V(x)$ is a control Lyapunov function and satisfies the bounded control property for the pair $(\pi, \widetilde{\alpha})$ then the dissipation inequality (11) is achievable for the system (12) using a control of the form $u = \psi(x)\pi(x)$ where ψ is nonnegative and as smooth as desired.*

---

[1]Here we use the Lie derivative notation which usually stands for $\frac{\partial V}{\partial x}(x)g(x)\pi(x)$ but our results apply to the case where $V(x)$ is locally Lipschitz and we use the more general directional derivative of Clarke.

Subsequently, we applied this general result to specific $\mathcal{L}_\infty$ and $\mathcal{L}_2$ disturbance atttenuation control problems. In doing so, we were able to come up with new results on backstepping locally Lipschitz disturbance attenuation control laws, perhaps defined implicitly through locally Lipschitz equations, through perturbed integrators. We also applied our result to derive a dynamic, explicit control algorithm for linear systems with actuator saturation from a recently developed, implicit control algorithm involving the solutions to a family of algebraic Riccati equations.

2.3   Anti-windup control design

In [12], we extended the novel anti-windup algorithm we developed under a previous AFOSR grant to the case of linear systems with actuator saturation and exponentially unstable modes to permit stability on the region in the state-space steerable to the origin by a saturated feedback control. This set is restricted only in the directions of the exponentially unstable modes. We produced a control algorithm that was able to guarantee the following:

1. If the nominal control algorithm never saturates, the nominal control algorithm is not modified;

2. From all initial conditions in the set steerable to the origin by saturated feedback control, the closed-loop trajectories remain bounded;

3. The response with saturation converges to the nominal response without saturation when the nominal response without saturation converges to a trajectory corresponding to small inputs and small exponentially unstable modes.

More recently, in [13], we have further extended this algorithm to cope with simultaneous actuator magnitude and rate saturation and have applied it to a linearized model of an unstable fighter aircraft with actuator magnitude and rate limits. We have paid particular attention to how the anti-windup algorithm interacts with a pilot flying the aircraft in a manual flight control mode. We have illustrated very high performance capabilities under the action of our anti-windup control algorithm.

2.4   Output feedback control design

Because we cannot typically afford to accurately measure all state variables of a control system, output feedback control results are crucial for solving real-world problems. However, output feedback control problems for nonlinear systems are notoriously difficult because of the possibility of finite escape times that are unobservable at the output.

In [14] we considered multi-input, multi-output nonlinear control systems

$$
\begin{aligned}
\dot{x} &= f(x, u, \mu(t)) \\
y &= h(x, u, \mu(t))
\end{aligned}
\tag{16}
$$

with $\mu(\cdot)$ belonging to the set of measurable functions taking values in a compact set. In this setting we established the fundamental result that if the system (16) is semiglobally practically asymptotically stabilizable by uniformly completely observable (UCO) feedback then the system (16) is semiglobally practically asymptotically stabilizable by dynamic output feedback. The definition of uniformly completely observable (UCO) dynamic feedback, given next, at times implicitly constrains $\mu(t)$ in (16) to be sufficiently smooth, where sufficiently smooth has to do with the number of times the output needs to be differentiated to reconstruct the UCO function.

A function $\varphi(x, u, \mu)$ is said to be *uniformly completely observable* (UCO) with respect to the system (16) if it can be expressed as a function of a finite number of derivatives of the output y and the input u, i.e., if there exist two integers n_y and n_u and a function Ψ such that, for each solution of

$$
\begin{aligned}
\dot{x} &= f(x, u, \mu(t)) \\
u^{(n_u+1)} &= v \\
y &= h(x, u, \mu(t))
\end{aligned}
\tag{17}
$$

we have, for all t where the solution makes sense,

$$
\varphi(x(t), u(t), \mu(t)) = \Psi\left(y(t), \ldots, y^{(n_y)}(t), u(t), \ldots, u^{(n_u)}(t)\right)
\tag{18}
$$

where $y^{(i)}$ denotes the ith time derivative of y at time t (and similarly for $u^{(i)}$).

Theorem 15 of [14] states the following:

Theorem 6 *Let $\mu(\cdot)$ belongs to the set of measurable a functions uniformly bounded and sufficiently smooth with a uniform bound on an appropriate number of its derivatives. If the origin of (16) is uniformly semiglobally practically asymptotically stabilizable by dynamic UCO feedback then it is uniformly semiglobally practically asymptotically stabilizable by dynamic output feedback.*

Another main result of [14] was to show that a special case of theorem 6 provides a control algorithm for semiglobally practically asymptotically stabilizing a large class of detectable, nonminimum phase nonlinear systems.

In [15], the output feedback problem for a restricted class of nonlinear systems was considered where the nonlinearities depend on the output. There it was shown that local optimality and (semi)global inverse optimality could be combined through a clever modification of standard backstepping algorithms.

2.5 Fundamental control limitations

In [16], we contributed to results that address fundamental control limitations and the development of control algorithms that push us to the boundary of these limitations. In [16],

we studied systems of the form

$$
\begin{aligned}
\dot{z} &= Fz + \Psi(z, \xi, u)u \\
\dot{\xi} &= A\xi + Bu
\end{aligned}
\tag{19}
$$

where (A, B) is stabilizable, F is Hurwitz with $\max Re\lambda(F) =: -\alpha < 0$ and $\max Re\lambda(A) =: \nu > 0$ and where there exist positive constants p, q and C such that, for z sufficiently small,

$$
|\Psi(z, \xi, u)| \le C|z|^{q+1}|(\xi, u)|^{p-1} .
$$

We showed that, in the absence of extra assumed structure, a structural limitation for being able to control the system (19) to the origin was the condition

$$
\frac{\nu}{\alpha} < \frac{q}{p} .
\tag{20}
$$

More significantly, when the condition (20) is satisfied, we were able to explicitly construct a smooth feedback law rendering the origin of the system (19) globally asymptotically stable.

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APPLICATION

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4   Personnel

Faculty partially supported :   Andrew R. Teel (P.I.)
Students supported: Corneliu Barbu (Ph.D. candidate)

5   Awards

1998 IEEE Leon K. Kirchmayer Prize Paper Award.

> A.R. Teel "A nonlinear small gain theorem for the analysis of control systems with saturation", *IEEE Trans. on Automat. Contr.*, vol. 41(9), 1256-1270.

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